Nyquist criterion

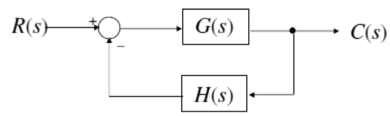
Nyquist diagram

What is Nyquist criterion about?

- Nyquist criterion allows us to determine the stability of the closed-loop system from a knowledge of the frequency-response of the open-loop function
- Nyquist diagram can be evaluated either mathematically if the open-loop function is available or experimentally if the physical system is available for experimentation (in the latter case, the open-loop system needs to be stable)
- Nyquist diagram gives us important information concerning the type of compensation required to stabilize certain types of systems

What is Nyquist criterion used for?

- In this lecture we consider the closed-loop systems of the following type



- Nyquist criterion is a unique method for determining stability of a closed-loop system
- Nyquist criterion allows us to determine the stability of a closed-loop system from the Bode diagram (or frequency response) of the open-loop function *G(jw)H(jw)*

What is Nyquist criterion used for?

- More about Nyquist: Can anyone name another

- contribution of Harry Nyquist? Nyquist rate (Sampling Theorem): For lossless digitization, the sampling rate should be at least twice the maximum frequency responses).

Cauchy's principle of argument

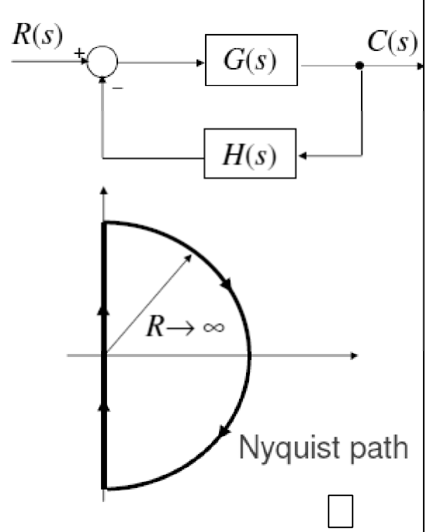
Let *F(s) be the ratio of two polynomials in s. Let the* closed curve *C in the s-plane be mapped into the* complex plane through the mapping *F(s). If F(s) is* analytic (complex differentiable) within and on *C,* except at a finite number of poles, and if *F(s) has* neither poles nor zeros on *C, then*

$$N = Z - P$$

where *Z is the number of zeros of F(s) in C, P is the* number of poles of *F(s) in C, and N is the number of* encirclements of the origin, taken in the same sense as *C.*

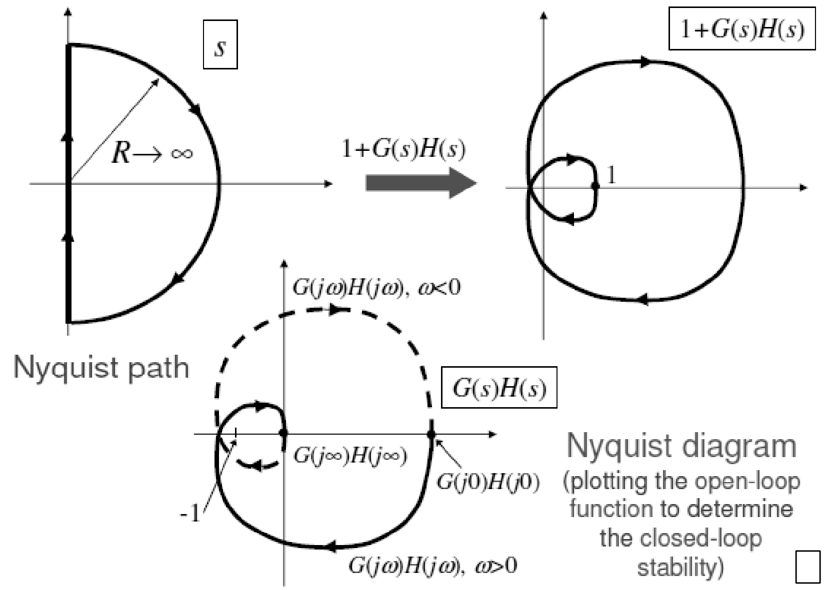
Nyquist diagram

Consider a mapping $F(s) = 1 + G(s)H(s)$ and a curve C composed of the imaginary axis and an arc of infinite radius such that the curve completely encloses the right half of the s-plane



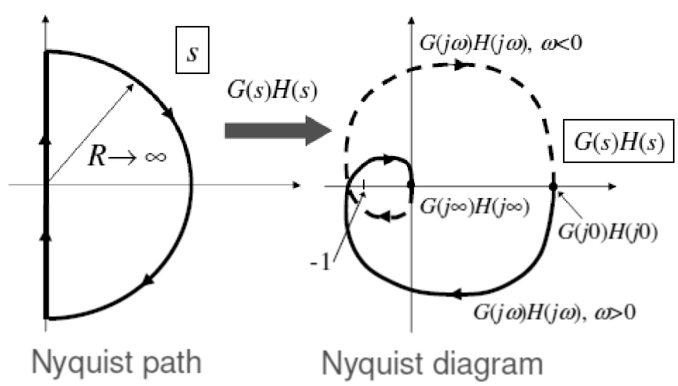
Question: For what value of Z (number of zeros of $F(s)$ in C) will the closed-loop system be stable?

Nyquist diagram



Nyquist diagram
(plotting the open-loop function to determine the closed-loop stability)

Nyquist criterion



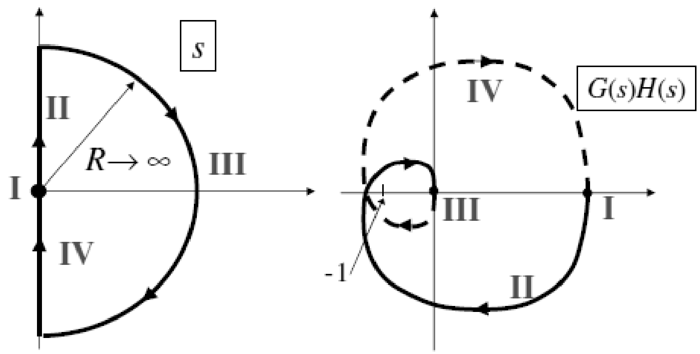
The Nyquist path is shown in the left figure. This path is mapped through the open-loop function $G(s)H(s)$ into the Nyquist diagram, as illustrated in the right figure. Then

$$N = Z - P$$

where Z is the number of roots of the system characteristic equation in the right half-plane, N is the number of clockwise encirclements of the -1 point, and P is the number of poles of the open-loop function $G(s)H(s)$ in the right half-plane.

Four parts of Nyquist path

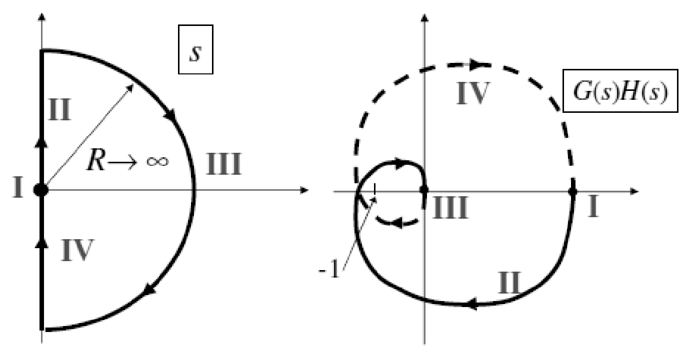
- Part **I**: the origin of the s-plane, which corresponds to $G(0)H(0)$ (or dc gain of the open-loop function) in Nyquist diagram



Nyquist path Nyquist diagram

Four parts of Nyquist path

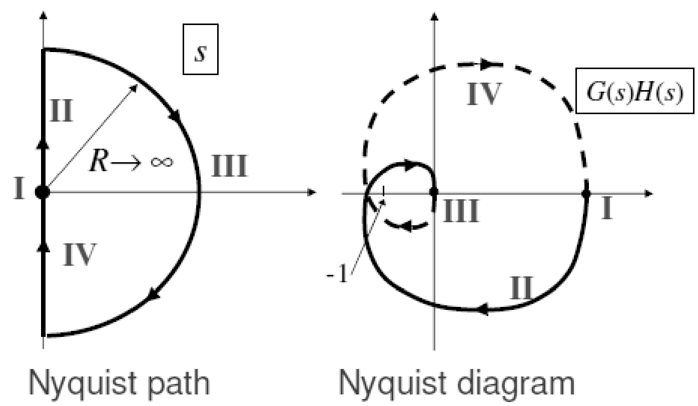
- Part **II**: the positive half of the imaginary axis, which corresponds to $G(j\omega)H(j\omega)$, $\omega > 0$ (frequency response of the open-loop function) in Nyquist diagram



Nyquist path Nyquist diagram

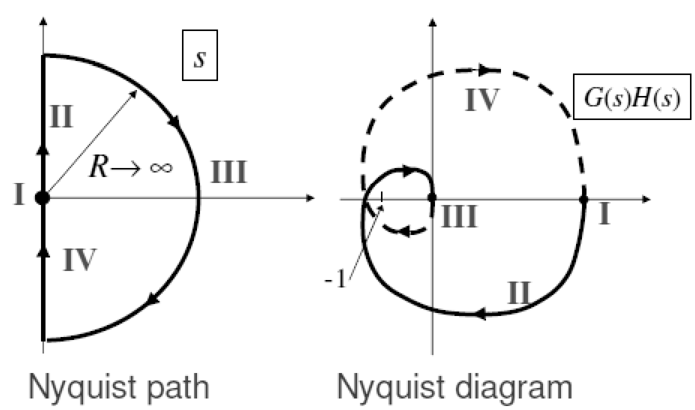
Four parts of Nyquist path

- Part **III**: the infinite arc. Since physical systems are low-pass in nature, the open-loop function evaluated along this arc is zero



Four parts of Nyquist path

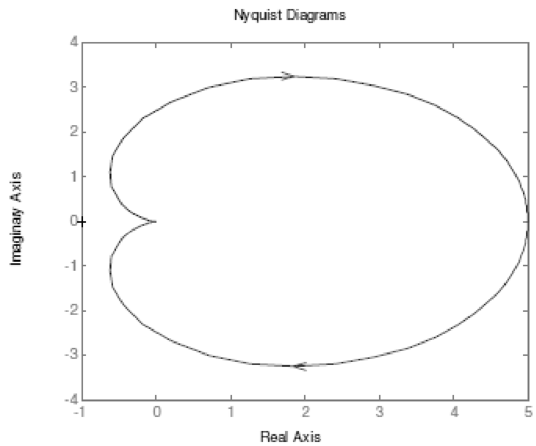
- Part **IV**: the negative half of the imaginary axis, which corresponds to $G(j\omega)H(j\omega)$, $\omega < 0$ (the complex conjugate of the open-loop function evaluated along Part **II**) in Nyquist diagram



Examples of Nyquist diagram

- Example 1:

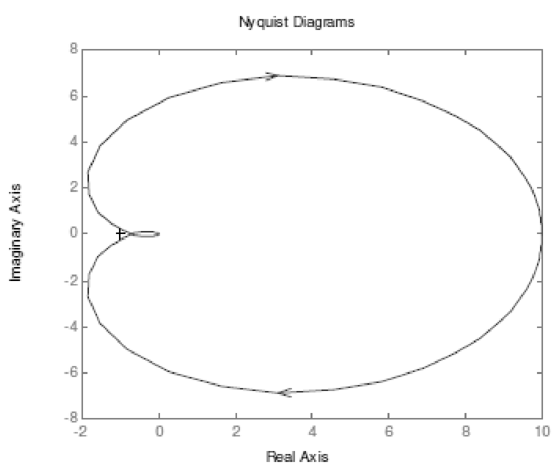
$$G(s)H(s) = \frac{5}{(s+1)^2}$$



Examples of Nyquist diagram

- Example 2:

$$G(s)H(s) = \frac{50}{(s+1)^2(s+5)}$$



- Time delay is one of the most difficult control problems to overcome
  - Appreciable time delay can cause the controller to become “impatient” (because of the delay, the controller may assume that its initial control efforts had no effect and that it needs to try harder to force a change)
  - Do you still remember the root loci of a system with an ideal time delay?

- A special case: poles at the origin
  - Consider the case where the open-loop function has a pole or multiple poles at the origin, for example

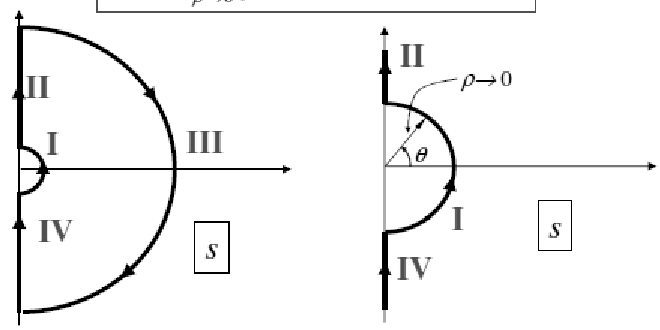
$$G(s) = \frac{50}{s(s+5)}$$

  - The procedure for analyzing this case also applies to the case of marginally stable open-loop systems with poles on the *jw-axis not equal to zero*

Poles at the origin

- A special case: poles at the origin
- Procedure for analyzing this case
  - Reform the Nyquist path to detour around the origin if the open-loop function has a pole there
  - The detour is chosen to be circular with a radius that approaches zero in the limit

$$s = \lim_{\rho \to 0} \rho e^{j\theta}, \quad -90^{\circ} \le \theta \le 90^{\circ}$$



Poles at the origin

- A special case of poles at the origin
- Procedure for analyzing this case
- Exercise 1

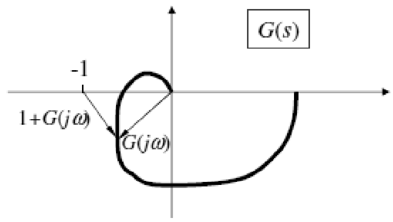
$$G(s) = \frac{K}{s(s+1)}, \quad H(s) = 1$$

- Exercise 2

$$G(s) = \frac{K}{s^2(s+1)}, \quad H(s) = 1$$

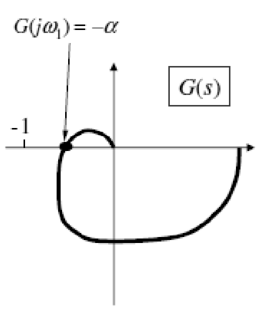
Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the closeness of the Nyquist diagram to the -1 point in the complex plane



Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the *closeness* of the Nyquist diagram to the -1 point in the complex plane



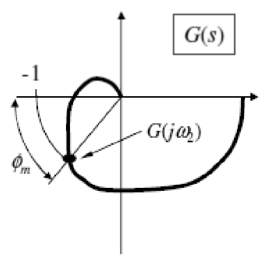
- Two measures of relative stability
  - Gain margin: If the magnitude of the open-loop function of a stable closed-loop system at -180^o crossover on the Nyquist diagram is the value α, the gain margin is $1/\alpha$
    - The margin is usually given in decibels
    - Phase crossover frequency is defined as the frequency where the phase shift of $G(j\omega)H(j\omega)$ is -180^o (in other words, the gain margin is defined at the phase crossover frequency)

15

Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the *closeness* of the Nyquist diagram to the -1 point in the complex plane

- Two measures of relative stability
  - Gain margin
  - Phase margin: the magnitude of the minimum angle (ϕ_m) by which the Nyquist diagram must be rotated in order to intersect the -1 point for a stable closed-loop system
    - Gain crossover frequency is defined as the frequency where the magnitude of $G(j\omega)H(j\omega)$ is 0 dB or 1 in absolute value (in other words, the phase margin is defined at the gain crossover frequency)



16